Problem 1 (12 points)

(Courtesy J. Moran)

At millimeter and centimeter wavelengths, most extragalactic radio sources are time-variable and cannot be used reliably as calibrators. However, planets can be good sources for calibration purposes.

A) Mars has a brightness temperature of about 230 K (uniform over its disk) over this wavelength range. Is this reasonable, assuming Mars has no internal heat sources, i.e., its surface is in radiation equilibrium with the sun ($T_\odot = 5800$ K)? (4 points)

Solar luminosity: $L_\odot = \sigma T_\odot^4 4\pi R_\odot^2$

Solar flux at Mars distance: $F_{\odot,Mars} = \frac{1}{4\pi d_{Mars}^2} \sigma T_\odot^4 4\pi R_\odot^2 = \sigma T_\odot^4 \left( \frac{R_\odot}{d_{Mars}} \right)^2$

Total energy/time absorbed by Mars: $\sigma T_\odot^4 \left( \frac{R_\odot}{d_{Mars}} \right)^2 * \pi R_{Mars}^2$

Energy/time radiated by Mars: $L_{Mars} = \sigma T_{Mars}^4 4\pi R_{Mars}^2$

Energy balance:

$$L_{\odot,Mars} = L_{Mars}$$

$$\sigma T_\odot^4 \left( \frac{R_\odot}{d_{Mars}} \right)^2 \pi R_{Mars}^2 = \sigma T_{Mars}^4 4\pi R_{Mars}^2$$

$$T_{Mars}^4 = \frac{1}{4} T_\odot^4 \left( \frac{R_\odot}{d_{Mars}} \right)^2$$

Plug in numbers...

$T_\odot = 5800$ K

$R_\odot = 7 \times 10^8$ m

$d_{Mars} = 2.3 \times 10^{11}$ m

$\Rightarrow T_{Mars} = 230$ K

Yes, Mars is in radiative equilibrium with the Sun.
B) What is the flux density of Mars at $\lambda = 0.87 \text{ mm}$ wavelength on February 13, 2013? You can find the angular size of Mars on the SMA Observer Center website (sma1.sma.hawaii.edu; choose “Tools” and “planetary visibility function calculator”). (4 points)

Flux density $S_\nu = \frac{2E_B c^2}{\lambda^2} \Omega$; substitute $\frac{c^2}{\lambda^2} = \frac{1}{\lambda^2}$; observing at $\lambda = 8.7 \times 10^{-2} \text{ cm}$

Diameter of Mars is $4''$, so $\Omega = \frac{\pi}{4} \theta_M^2 = \frac{\pi}{4} \left( \frac{4}{180 \times 3600} \right)^2 = 3 \times 10^{-10} \text{ Sr}$

Then $S_\nu = \frac{2(1.4 \times 10^{-14} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ K}^{-1}) \times (2.30 \text{ K})}{(8.7 \times 10^{-2} \text{ cm})^2} \times 3 \times 10^{-10} \text{ Sr} = 2.6 \times 10^{-21} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1} = 260 \text{ Jy}$

C) Assume that we observe Mars with one of the 6 m diameter SMA antennas at 0.87 mm. What would the antenna temperature be for the point source approximation? (4 points)

Point source: $T_a = T_B \frac{\Omega_s}{\Omega_a}$

$T_B = 230 \text{ K}$, and $\Omega_s = 3 \times 10^{-10} \text{ Sr}$ (from part B above)

Antenna beam solid angle: $\Omega_a = \frac{\pi}{4} \left( \frac{\lambda D}{6 \times 10^{10} \text{ cm}} \right)^2 = 1.7 \times 10^{-8} \text{ Sr}$

Then $T_a = 230 \text{ K} \frac{3 \times 10^{-10} \text{ Sr}}{1.7 \times 10^{-8} \text{ Sr}} = 4 \text{ K}$

Problem 2 (25 points)

We will be using an IBT (Itty-Bitty Telescope) to explore the 12 GHz sky around the Wesleyan campus. The telescope has a diameter of $\sim 1 \text{ m}$ (slightly less). Please estimate the antenna temperature of the following celestial or terrestrial objects, and rank them from highest to lowest. (Consider whether this is what you would expect observing the same objects with an optical telescope.) Are there other objects in the sky that you might expect to be bright at 12 GHz? (Ranking/discussion: 5 points)

- **The Cosmic Microwave Background (2 points)**
  When the source fills the beam, $T_a = T_b = 3 \text{ K}$

- **The Sun (2 points)**
  At 12 GHz, the sun is still dominated by its 5800 K blackbody component (plus a little extra, but we’ll ignore that for now).
  Solar diameter is $\theta_\odot \approx 0.5^\circ$ ⇒ $\Omega = \frac{\pi}{4} \theta_\odot^2 = 6 \times 10^{-5} \text{ Sr}$
  Beam size of telescope is $\Omega_a = \frac{\pi}{4} \left( \frac{\lambda}{D} \right)^2 = 5 \times 10^{-4} \text{ Sr}$
  Antenna temp: $T_a = T_B \frac{\Omega_s}{\Omega_a} = 700 \text{ K}$

- **A nearby star (2 points)**
  As in class, assume $T_{phot} = 5000 \text{ K}$, radius = 0.005 AU ($\sim R_\odot$), distance = 10 pc
  $\Omega_s = \left( \frac{5 \times 10^{-3} \text{ AU}}{10 \text{ pc}} \right)^2 = 6 \times 10^{-18} \text{ Sr}$
  Antenna temp: $T_a = 5000 \text{ K} \frac{6 \times 10^{-18}}{5 \times 10^{-14}} = 6 \times 10^{-11} \text{ K}$

- **The Crab Nebula supernova remnant, which has a flux of $\sim 500 \text{ Jy}$ at 12 GHz, and a size of 7×5 arcmin. (2 points)**
  Recall: $T_b = \frac{L_v c^2}{2 k T}$ and $I_v = \frac{F_v}{4 \pi}$
  Assuming a round remnant with diameter 6', calculate $\Omega_s = 2.4 \times 10^{-6} \text{ Sr}$
  $I_v = \frac{500 \times 10^{-23} \text{ erg cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}}{2.4 \times 10^{-15} \text{ erg cm}^{-2} \text{ Hz}^{-1} \text{ Sr}^{-1}} = 2.1 \times 10^{-15} \text{ erg cm}^{-2} \text{ Hz}^{-1} \text{ Sr}^{-1}$
$T_b = \frac{2.1 \times 10^{-15} \frac{\text{erg}}{\text{cm}^2 \text{Hz} \text{Sr}}} {2 \nu^2} = 47 \text{ K}$

$T_a = 47K \frac{2.4 \times 10^{-6} \text{Sr}} {3.5 \times 10^{-15} \text{Sr}} = 0.2 \text{ K}$


  3C48 is 2.2 Jy

  $T_a = \frac{S_{\text{nu}} A_{\text{eff}}}{2\nu k} = \frac{2.2 \times 10^{-21} \frac{\text{Jy}}{\text{Hz}} \pi (100 \text{ cm})^2}{2 \times 1.38 \times 10^{-26} \frac{\text{erg}}{\text{cm}^2 \text{Hz} \text{Sr} \pi^2 (100 \text{ cm})^2}} = 6 \times 10^{-4} \text{ K}$

  (You can think about how the antenna temperature of a point-like quasar will change relative to the CMB as you vary the size of your telescope.)

- The planet Saturn (assume radiation equilibrium as in Problem 1) (2 points)

  Distance to Saturn is $\sim 10$ AU

  $T_{\text{Sat}} = T_{\odot} \left( \frac{R_{\odot}}{2d_{\text{Sat}}} \right)^{1/2} = 5800 \text{ K} \left( \frac{2 \times 1.5 \times 10^{13} \text{ m}}{d_{\text{Sat}}} \right)^{1/2} = 90 \text{ K}$

  Diameter of Saturn is $\sim 17''$ ⇒ $\Omega_{\text{Sat}} = \frac{\pi \theta_{\text{Sat}}^2} {4} = \frac{\pi (\frac{17}{180} \text{ radian})^2} {4} = 5 \times 10^{-9} \text{ Sr}$

  $T_a = T_b \frac{\Omega_{\text{Sat}}} {\Omega_a} = 90 \frac{5 \times 10^{-9}} {6 \times 10^{-7}} = 9 \times 10^{-4} \text{ K}$

- VVO (assume we are observing from the parking lot) (2 points)

  VVO is large enough to fill the $\sim 2.5^\circ$ beam. Its brightness temperature will be roughly the ambient temperature (with some adjustment for emissivity), or $\sim 280 \text{ K}$.

- Your fist, held right in front of the dish (2 points)

  Since we’re so close to the telescope, here we can substitute the fractional area of the aperture covered by your hand for the ratio of solid angles.

  I measured the area of my fist to be $\sim 40 \text{ cm}^2$. Let’s assume that my fist is a little warmer than ambient temperature, say 300 K.

  $T_a \sim 300 \frac{40 \text{ cm}^2} {1 \times 10^{10} \text{ cm}^2} \sim 1 \text{ K}$

- Your hand, held right in front of the dish (2 points)

  The covered by my hand is $\sim 2 \times$ the area covered by my fist, so $T_b$ will double to $\sim 2 \text{ K}$. So the brightness temperature of your hands approaches the brightness temperature of the CMB. (That’s why waving your hands/arms in front of the aperture can produce a response against the background of the CMB, but not against the background of the building.)

- An unladen European swallow flying 10m overhead (2 points)

  The average wingspan of a swallow is something like 30cm (we’ll approximate it as a square object), so it will cover a solid angle of $\sim \frac{\pi}{4} \left( \frac{30 \text{ cm}} {10 \text{ cm}} \right)^2 = 7 \times 10^{-4} \text{ Sr}$ i.e., it will approximately fill the beam.

  Giving it a $T_b$ of 300 K (does anybody have a better guess for the skin temperature of a bird?) its antenna temperature is then simply $\sim 300 \text{ K}$.

So our list, ranked by antenna temperature, becomes:

- Sun
- Bird / VVO (both approximately fill the beam at roughly ambient temperature)
- CMB
- Your hand/fist (larger portions of your body, like an arm, can trump the CMB)
• Supernova remnant

• Saturn / quasar (similar $T_a$ in this telescope – how will this vary with diameter/frequency?)

• And, WAY out in last place, is a star. Stars are very difficult to observe with radio telescopes (except through nonthermal processes like flares and magnetic interactions).

The galactic synchrotron emission is also quite bright at radio wavelengths, although by the time the frequency gets as high as 12 GHz it is no longer the brightest source. Some of the brightest objects in the 12 GHz sky are satellites (which commonly transmit at this frequency). It’s not too difficult to find geosynchronous satellites with the IBT.