Problem 1

(Courtesy J. J. Condon & S. M. Ransom)
The spin-down relation in terms of period $P$ is $\dot{P} \propto P^{2-n}$.

A) Use this relation (assuming an initial spin period $P_0$) to show that the pulsar age $T$ is

$$T = \frac{P}{(n-1)\dot{P}} \left[ 1 - \left( \frac{P_0}{P} \right)^{n-1} \right]$$

(1)

without assuming magnetic dipole braking, constant magnetic field strength, or $P_0 \ll P$.

B) Show that for the common assumptions of $n = 3$ and $P_0 \ll P$ this reduces to the characteristic age, $\tau_c$, as derived in class.

C) If a 100ms pulsar is found with $\tau_c = 30$ kyr in a supernova remnant where historical or kinematic data suggest a true age of only $\sim$2 kyr, what does this imply about the pulsar? Can this discovery constrain the braking index? Why or why not?

Problem 2

(Courtesy M. Faison)

A) What is the fringe spacing $\theta_f$ for a two-element, east-west interferometer with a projected baseline of 230 m observing at a frequency of 5 GHz?

B) Two point sources are located in the primary beam of the interferometer 12 arcseconds west of the center and 6 arcseconds east of the center. The west source has a flux that is double that of the east source (call the flux of the east source “$F$”). What are the visibility amplitude and phase measured at this instant by the interferometer?
Problem 3
(Adapted from J. Moran)
Early attempts to measure superluminal motions in quasars were based on very sparse $(u,v)$ plane data. Whitney et al. (Science, 173, 225, 1971; see figure) observed the quasar 3C279 at 4 cm wavelength with a VLBI (Very Long Baseline Interferometer) on October 14, 1970 and February 14, 1971. Their projected baselines were limited to the range of 200-400 cycles per arcsecond. (Note that the natural unit of $u$ and $v$ is cycles per radian.) In October they found a null in the fringe visibility at $u = 320$ cycles/arcsecond and in February at $v = 290$ cycles/arcsecond. They fit these data to a model of a double source with changing component separation. The diameters of the antennas are 37 and 64 m, respectively.

A) Convince yourself (and me) that the response, $V$, of the interferometer to a double source with component separation of $a$ and equal brightness temperatures $b$ is

$$|V(u,v)| = 2b \frac{\Omega_x}{\Omega_u} |\cos \pi u a|$$  \hspace{1cm} (2)

B) If the peak flux density is 4 Jy (note that 1 Jy used to equal 1 flux unit!), what is the lower limit on the estimate of the brightness temperature of the components if they are unresolved?

C) Show that with this model $a$ varied from 0.00155" to 0.0017".

D) Calculate the apparent velocity of separation based on the distance of $2 \times 10^9$ pc.

E) As an alternative to this model, show that the data can be explained if in October there was an additional source having a brightness temperature $b'$ ($b' \ll b$) midway between the other two components that disappeared by February. What might $b'$ have been?