Problem 1

(Courtesy J. Moran)
Suppose you are given the option of searching for a point radio source (e.g., a Gamma Ray Burst, or GRB) of uncertain position with a big dish of diameter $D$ that has only one receiver, or with an interferometric array of $N$ elements of diameter $d$, with maximum baseline $D$. Assume that the source size is $< \lambda/D$ and its position uncertainty is $\pm \lambda/d$. Also, assume that the instruments have the same bandwidth and receiver temperatures. If you can have the same amount of total integration time on either instrument, which would you choose?

Problem 2

(Courtesy J. Moran)
Let’s explore the performance of radiometers that have detectors that do not have a “square law” characteristic, i.e., $v_3 \neq v_2^2$. Consider a radiometer with a linear detector instead of a square law detector, as shown below:

![Radiometer Diagram]

Assume:
$v_3 = |v_2|$
$< v_2 > = 0$
$< v_2^2 > = kBG(T_R + T_A)$
Also assume that the signals are weak ($T_A \ll T_R$).
A) Show that

\[ < v_4 > = \sqrt{\frac{2}{\pi} kBG(T_R + T_A)} \]  

(1)

and hence that the term representing the signal is

\[ \approx T_A \sqrt{\frac{kBG}{2\pi T_R}} \]  

(2)

B) Show that the rms noise, \( \sigma_4 \), is

\[ \approx \sqrt{\frac{(1 - \frac{9}{2}) kBG T_R}{2 B \tau}} \]  

(3)

and that the radiometer sensitivity given by the equation

\[ \Delta T = \frac{\sigma_4}{\frac{\partial < v_4 >}{\partial T_A}} \]  

(4)

is

\[ \Delta T = \frac{1.07 T_R}{\sqrt{B \tau}} \]  

(5)

Hence, this type of receiver is 7% worse than the square law detector receiver, and is linear in power only at low signal levels.
Problem 3

(Courtesy J. Moran)

I took the following calibration data on one of the antennas of the VLA in order to estimate the receiver temperature and the atmospheric opacity: The sources are weak enough that their contribution to the system temperature is negligible. Assume that the atmosphere is stable with time and can be modeled as a plane parallel absorbing medium. The calibration temperature ($T_{Cal}$) is 12.5 K ($T_{Sys} = T_{Rx} + T_{Atmosphere}$). Estimate the receiver temperature and the zenith opacity.

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<th>Time (UT)</th>
<th>Source</th>
<th>Elevation (°)</th>
<th>$T_{Sys}/T_{Cal}$</th>
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