ASTR240: Radio Astronomy

HW#6

Due May 8, 2013

Problem 1

(Courtesy J. Moran)
A brilliant engineer named D. Donut decides that the center of a large parabolic antenna is not of much use because, for one thing, the resolution or beam size is approximately equal to λ/D, where D is the overall diameter of the antenna and λ is the wavelength. He proposes to build an antenna that is a ring section of a parabola (i.e., a parabolic reflector without the center section) with thickness ΔD and diameter $D \gg \Delta D$. A special feed uniformly illuminates the reflector. He claims that the beam width will be equal to that of a normal parabolic antenna of almost twice the diameter!

A) Estimate the gain function of this antenna, $G(\theta, \phi)$ (No hairy integrals, please!). What is the beam width, θ_A?

B) What is the effective on-axis collecting area, A_0?

C) What is the antenna beam solid angle, Ω_A?

D) What are the advantages and disadvantages of this antenna?

FYI: the Culgoora Array in Australia, now defunct, consisted of a large number of paraboloid antennas arranged in a circle and had an antenna pattern something like the one discussed here.

Problem 2

(Courtesy J. Moran)
An interferometer having an east-west orientation and baseline length D is used to map a source near the celestial pole so that the u, v plane track is a circle. Show that the equivalent beam pattern will be circularly symmetric and given by

$$G_1(\theta) \quad \propto \quad J_0\left(\frac{2\pi D\theta}{\lambda}\right) \tag{1}$$

(2)

where $\theta = \sqrt{\alpha^2 + \beta^2}$ and α and β are angles in the direction of RA and dec.

A) If all the spacings shorter than D are measured by shortening the baseline, show that

$$G_2(\theta) \propto \frac{2J_1(2\pi D\theta/\lambda)}{2\pi D\theta/\lambda} \tag{3}$$

Interferometer maps frequently have negative temperatures because of the negative sidelobes of these patterns.

B) What is the reason that G_1 is not the same as that of the ring antenna in Problem 1?

C) Why is G_2 not the same as an Airy pattern?